The Impact, Rebound and Flight of a Well Inflated Pellicle as Exemplified in Association Football

By
W Johnson, S R Reid and R E Trembaczowski-Ryder

Presented on 23rd March, 1973
to the Manchester Association of Engineers

Introduction

It is curious that little or no scientific study of the impact and rebound of a well inflated, inextensible shell, e.g. a football, exists. The elastic and plastic impact of solid spheres has long been a subject of very detailed investigations so that the term 'Hertzian stress' is well known, yet comparable analyses concerning tile impact of inflated thin spherical pellicles is hardly to be found. The flight of a sphere - a bluff body - through air has, however, been the subject of many researches and references [1] to [4] are typical. This is not to say, however, that there is a plethora of theoretical and experimental results on hand which can be applied, for there are gaps in our fundamental knowledge of the subsonic aerodynamics of flow over bluff bodies.

In what follows an endeavour is made to describe and discuss quantitatively the two aspects of ball or spherical shell mechanics to which the mechanical sciences can contribute.

These are,

(i) The normal impact and rebound of a typical present-day football from a rigid plane surface assuming the ball to possess rectilinear motion only (no spinning) and that there are no hysteresis or other energy losses .

A linearised theory is first proposed which assumes that (a) the depth of the deformation zone is small relative to the radius, and (b) the pressure inside the ball remains constant throughout impact; a relationship is established between the maximum area of contact and the impact speed and also an estimate is made of the time of contact. The two simplifications (a) and (b) are removed in turn and the effects of doing so on the relationships between the various parameters are calculated. The theoretical results are compared with those of a series of experimental tests performed over a limited range of impact velocities and are shown to be in good agreement.

To the authors' knowledge there is no Information on this subject. Their contribution is therefore novel, albeit in some respects oversimplified and provisional.

(ii) The phenomena associated with the flight of the ball can be understood easily by the layman when once the concepts of drag and lift are appreciated. Every engineering student will have dealt with them at some time or other during his academic training. Their application to the movement of a football in a gravitational field, however, is not straightforward but meaningful results or calculations can be made with the aid of permissible assumptions, In this second section the topic is surveyed and a number of suggestions made. There is need for direct experimental verification by observing the game of football itself, probably using a fast camera.

The authors' aims in this paper are to present some new results, to furnish engineering science teachers with attractive examples which are close to students' interests, to stimulate further detailed investigations of the several absorbing problems surrounding this topic after exposing the limitations of the approach and to enrich the appreciation of the game of football.

NOTATION

d	diameter of ball
e	equatorial speed/wind speed
f	rectilinear acceleration of ball
m_1	mass of free portion
m_2	mass of contact portion = $M - m_1$
p	current internal pressure in excess of atmospheric pressure
P_A	atmospheric pressure
P0	initial internal pressure in excess of atmospheric pressure current radius of
r	contact area
r_m, y_m	maximum values of r & y
z_m, A_m	maximum values of z & A
t	time
u	current flight speed of the ball
u_c	speed of ball at critical Reynolds number
u_0	impact speed
v	dz/dt
v_o	u_o/R
x	distance from rigid plane to G
y	current polar height of depression in pellicle (see Fig 2)
z	y/R
A'	presentation area of ball πR^2
A	current area of contact
C_D	drag coefficient
C_L	lift coefficient
D	drag force
G	centroid of deformed pellicle
G_1	centra id of free portion of pellicle
G_2	centroid of contact portion of pellicle
L	lift force
M	mass of pellicle
O	centre of undeformed pellicle
R	radius of pellicle
Re	Reynolds number
T	time of contact
V	current volume of pellicle
V_o	volume of undeformed pellicle $4\pi R^3/3$
α	$2\pi R p_o/M$
β	$C_D \rho A'/2M$
γ	ratio of specific heats for air, taken to be 1.4
ν	kinematic viscosity of air
ρ	density of air
$\Delta\Theta$	increase in temperature of air within pellicle

Θ_0 initial temperature of air within pellicle

θ angle subtended at centre of a circle

PART 1. IMPACT AND REBOUND OF A WELL-INFLATED PELLICLE

The analysis of the dynamics of the rebound of the pellicle from a frictionless, plane, rigid surface is based upon certain simplifying assumptions regarding the mode of deformation of the sphere during the time of contact and the nature of the material comprising the pellicle. In as far as the properties of the material i.e. elastic or otherwise, do not enter into the analysis, the problem is treated here simply as one in dynamics; it is at no time a hyperstatical one.

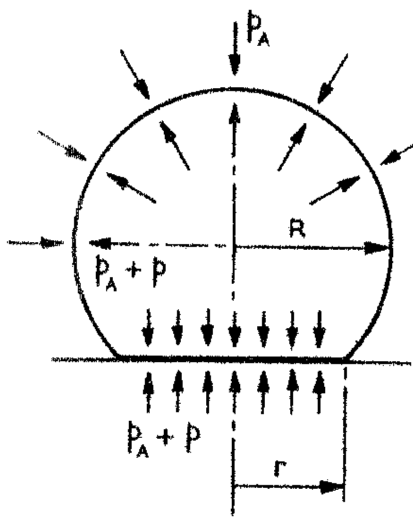

Fig 1
Assumed profile of deformed pellicle

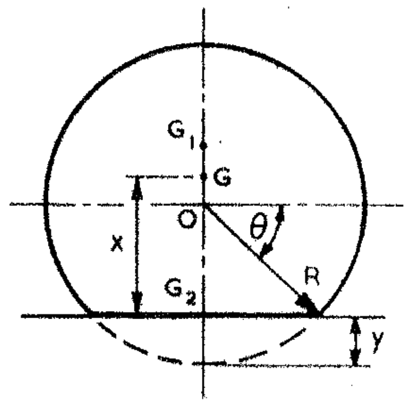

Fig 2
Centroids of deformed pellicle and its component parts

It is assumed that during the period of contact the shape adopted by the membrane is as shown in Fig 1. The contact area is a disc of current radius r whilst the remainder of the membrane (Called the free portion) is spherical of radius R, the pre-impact radius. The material is therefore considered to be virtually inextensible since the radius does not respond by any appreciable amount to the change of internal pressure consequent on the decrease in the volume enclosed by the membrane. The specification that the contact zone is flat neglects any tendency of that part of the pellicle to buckle inwards. A detailed analysis of the conditions under which the mode of deformation changes from that shown in Fig 1 to one in which all or part of the contact portion has buckled is beyond the scope of the present paper. In view of the fact that the membrane stresses are initially tensile due to inflation and that the contact zone is under the influence of the internal pressure inside the pellicle it is expected that buckling will be inhibited for a considerable range of impact velocities.

The contact pressure is taken to be uniform over the disc of radius r and of value $(p_A + p)$ where p is the current internal pressure in excess of the atmospheric pressure p_A. No extra line load around the periphery of the contact area is included as the flexural rigidity of the pellicle is also neglected. In order to specify this latter load and to examine the buckling effect it would be necessary to use non-linear shell theory, see e.g. [5].

By considering the pellicle to be a system of particles of total mass M, the equation of motion can be deduced. In Fig 2, O is the position of the point which was the centre of the pellicle before impact after it has moved a distance y beyond initial contact. G_1 is the centroid of the free portion which has mass m_1 and G_2 is the centroid of the contact portion which has mass m_2 $(=M - m_1)$. From the geometry of Fig 2 and the definition of the centroid of a system of particles we have,

$$OG_1 = \frac{R}{2}(1 - \sin\theta) \text{ and } OG_2 = R\sin\theta \qquad (1)$$

$$m_1 = \frac{M}{2}(1 + \sin\theta) \text{ and } m_2 = \frac{M}{2}(1 - \sin\theta) \qquad (2)$$

If G represents the centroid of the deformed pellicle, then x, its distance from the plane of impact, can be found by taking moments about G. Thus

$$\frac{M}{2}(1 + \sin\theta)\left[\frac{R}{2}(1 - \sin\theta) + R\sin\theta - x\right] = \frac{M}{2}(1 - \sin\theta).\, x.$$

$$\text{Hence } x = \frac{R}{4}(1 + \sin\theta)^2 \qquad (3)$$

By considering the external forces (shown in Fig 1) acting on the pellicle, its equation of motion is

$$M\frac{d^2}{dt^2}\left[\frac{R}{4}(1 + \sin\theta)^2\right] = \pi r^2 (p_A + p) - \pi r^2 p_A = \pi r^2 p$$

or

$$M\frac{d}{dt}[(1 + \sin\theta)\cos\theta\, \theta'] = 2\pi R\cos^2\theta.\, p\,(\theta) \qquad (4)$$

since r = R Cosθ and allowing for the fact that the excess pressure may vary as the deformation proceeds. The notation θ' represents differentiation with respect to time. The degree of flattening of the pellicle is described by the paramter z = y/R. Since z = 1--sinh, equation (4) can be re-written in terms of z as

$$M\frac{d}{dt}[(2 - z)\, z'] = -2\pi R z\,(2 - z).\, p\,(z) \qquad (5)$$

The notation z' and z" represent differentiation with respect to time.

The remainder of this section is devoted to three solutions of equation (5).

First approximation

First consider the case in which the values of z and z' are sufficiently small that their squares and products can be neglected and that the pressure p(z) is effectively constant at its initial value of p_0 In this approximation equation {5) becomes

$$\frac{d^2z}{dt^2} = \frac{-2\pi R p_0}{M} z = -\alpha z \qquad (6)$$

putting $\alpha = 2\pi R p_0 / M$. According to this equation the motion of the pellicle during contact is simple harmonic of frequency $\omega = \sqrt{\alpha}$.

Integrating (6) gives

$$(z')^2 = \frac{u_0^2}{R^2} - \alpha z^2, \qquad (7)$$

where u_0 is the speed of the pellicle on impact, (i.e. the value of y' at t = 0). If $z = z_m$ when the pellicle comes instantaneously to rest, then from (7)

$$z_m = \frac{u_0}{R\sqrt{\alpha}} = u_0 \sqrt{\frac{M}{2\pi R^3 p_0}}. \qquad (8)$$

Since the motion is simple harmonic, the time of contact, T, is half the period

$$T = \frac{\pi}{\omega} = \frac{\pi}{\sqrt{\alpha}} = \sqrt{\frac{\pi M}{2Rp_0}}. \qquad (9)$$

Later, a comparison is made with the results of a set of tests which were performed on an Association Football in which the maximum area of contact was measured and correlated with the impact speed. For the purpose of comparison

$$A_m = \pi r_m^2$$

where A is the area of contact and the suffix m denotes the maximum values of the parameters. From Fig 1

$$r_m^2 = y_m (2R - y_m).$$

However, to be consistent with the linearisation of equation (5), (10) is used in the form $r_m^2 \simeq 2Ry_m$. Thus A_m =, $2\pi R y_m$.= $2\pi R^2 Z_m$ and therefore

$$A_m = u_0 \sqrt{\frac{2\pi RM}{p_0}}. \qquad (11)$$

Second approximation

For this case the effects of geometrical non-linearity are included by retaining all the terms in equation (5) but p(z) is again to be considered as constant at p_o. In this way, comparing the results of this approximation with those of a third approximation in which p(z) is allowed to vary, the significance of the compressibility of the air can be seen. The equation of motion is

$$\frac{d}{dt}[(2-z)\,z'] = -\alpha z\,(2-z) \qquad (12)$$

This equation can be integrated once using the integrating factor (2-z)z'. Thus

$$\tfrac{1}{2}(2-z)^2(z')^2 - 2v_0^2 = -\alpha\int_0^z w\,(2-w)^2 dw, \qquad (13)$$

where $v_0 = u_o/R$ and w is the dummy variable of integration. The relationship between u_0 and z_m can be found by setting z' = 0 and $z = z_m$ in equation (13). Hence,

$$u_0 = R\sqrt{\alpha}\, z_m\,(1 - \tfrac{2}{3}z_m + \tfrac{1}{8}z_m^2)^{\frac{1}{2}}. \qquad (14)$$

Since $A_m = \pi R^2 Z_m(2-z_m)$, from (10), the relationship between u_o and A_m can be established over a range of z_m.

The time of contact (and the value of z_m for a given u_0) can be found by integrating numerically the system of first order ordinary differential equations

$$\left.\begin{aligned} v &= z', \\ \text{and} \quad \frac{dv}{dt} &= -\alpha z + \frac{v^2}{2-z}, \end{aligned}\right\} \qquad (15)$$

with $v = u_0/R$, z = 0 at t = 0, which is equivalent to solving equation (11). This requires that α be specified numerically, a typical set of results will be presented and discussed later.

Third approximation

The assumption of constant pressure during the compression of the pellicle is now removed. Since the contact times both experimentally (see later) and according to the two theoretical approximations above are comparatively short, it is assumed that the compression of the air inside the pellicle takes place adiabatically. Hence

$$(p_A + p)\,V^\gamma = (p_A + p_0)V_0^\gamma, \qquad (16)$$

where p and V are the excess pressure and volume enclosed by the membrane and p_0 and V_0 are their values before impact. γ is the ratio of specific heats for air taken to be 1.4 in the example given later. According to the mode of deformation depicted in Fig 2, the decrease in the volume enclosed by the membrane, (V_0 - V), for a given value of y is the volume of the spherical cap of radius R and polar height y.

Thus, (Vo - V) = $^1/_3\pi y^2(3R - y) = {}^1/_3 oR^3z^2(3 - z)$.

Hence

Thus, $(V_0 - V) = \frac{1}{3}\pi y^2(3R - y) = \frac{1}{3}\pi R^3 z^2(3 - z)$.

Hence

$$\frac{V_0}{V} = \frac{1}{[1 - \frac{1}{4}z^2(3 - z)]} \quad \text{since } V_0 = \frac{4}{3}\pi R^3.$$

Substituting this into (16) and re-arranging,

$$p = \frac{(p_A + p_0)}{[1 - \frac{1}{4}z^2(3 - z)]^\gamma} - p_A.$$

For the case of an Association Football treated later, p_0= 15 Ibf/in^2 and in the example given, p_A will also have the same value. Thus (17) is simplified to

$$\frac{p}{p_0} = \frac{2}{[1 - \frac{1}{4}z^2(3 - z)]^\gamma} - 1 \qquad (18)$$

Values of p/p0 are given below in Table 1 for values of z over the range 0.1 to 1.0. It can be seen that the force which produces the deceleration and subsequent acceleration of the pellicle is substantially increased for the latter half of the range of z as compared with that given by the second approximation. Associated with this increase in pressure is an increase in temperature Q which can be evaluated using the equation of state,

$$\frac{(p_A + p)V}{(\Theta_0 + \Delta\Theta)} = \frac{(p_A + p_0)V_0}{\Theta_0}$$

$$\text{Hence } \frac{\Delta\Theta}{\Theta_0} = \left(\frac{p_A + p}{p_A + p_0}\right)\left(\frac{V}{V_0}\right) - 1 \qquad (19)$$

This is displayed in Table 1 for various values of z.

z	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
p/p_0	1.018	1.080	1.181	1.326	1.538	1.812	2.182	2.670	3.340	4.278
$\Delta\Theta/\Theta_0$	0.002	0.011	0.024	0.044	0.071	0.103	0.142	0.190	0.248	0.319

TABLE I

It is interesting to note that the final figure in the Table 1 corresponds to a temperature rise of about 100°C for an initial air temperature of about 40°C.

Inserting (17) into equation (5) a system of equations similar to (15) must be solved in which the second equation is now

$$\frac{dv}{dt} = -\alpha z \left(\frac{2}{[1 - \frac{1}{4}z^2 (3 - z)]^\gamma} - 1 \right) + \frac{v^2}{2 - z}. \qquad (20)$$

Unfortunately, unlike the second approximation, this cannot be integrated explicitly by using an integrating factor but a numerical solution can again be obtained for any given range of time. From this the time of contact and the relationship between u_o and A_m can be deduced.

Example: Rebound of an Association Football

The three solutions of the equation of motion described above may now be applied to the specific case of an Association Football. According to the Laws of the game, p_0 = 15 Ibf/in^2, M = 1 Ib, R = 0.36 ft, and these give α = 1.573 x 10^5 sec^{-2}. Thus according to the first approximation, the relationship between u_0 and A_m is given by

$$A_m = 5.7 \times 10^{-3} u_o \qquad ft^2,$$

if u_0 is given in ft/sec. This is shown in Fig 3 which also includes the results of a series of experiments to be discussed later

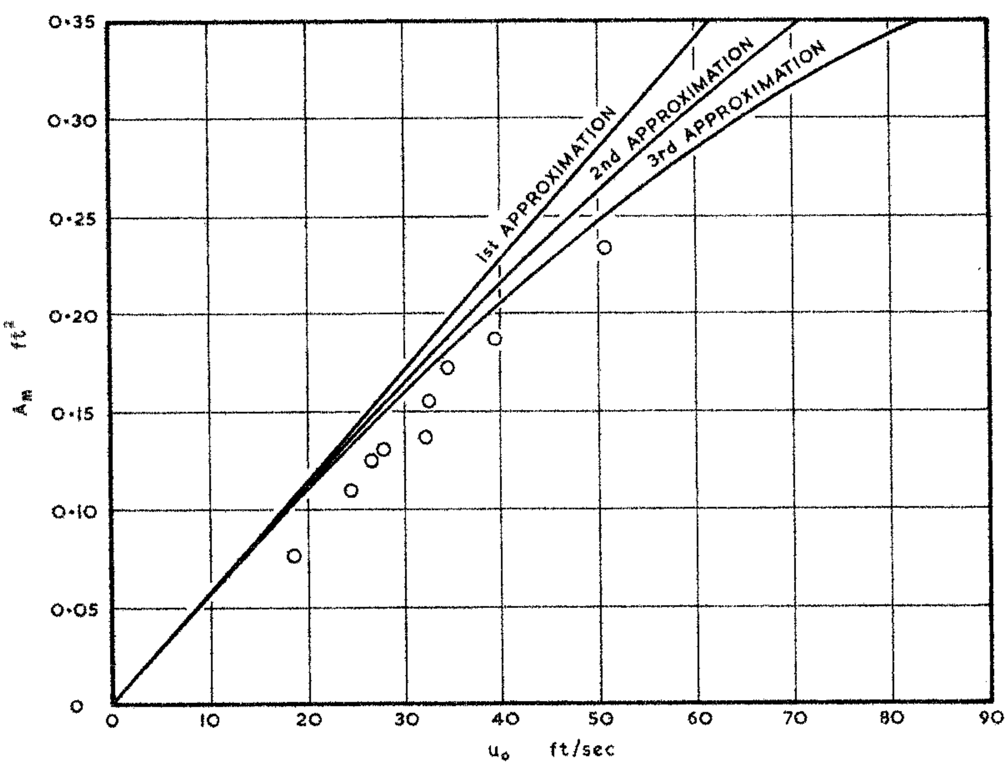

Fig 3 Maximum Area of Contact vs Impact Speed
---- Theoretical Curves 0 Experimental Points

According to equation (9) the first approximation predicts a constant time of contact of 7.92 rnillisec for the value of α given above whilst the second and third approximations indicate that T varies with u_o. The dependence of the time of contact on the impact speed is shown in Fig 4. All three approximations give a contact time of around 8 millisec up to an impact speed of about 20 ft/sec above which the second and third approximations diverge, the second approximation indicating an increase and the third approximation a decrease in T as u_o increases. This difference reflects the significant part played by the air pressure variation inside the ball at larger impact speeds.

Finally, an interesting feature of both the second and third approximations can be seen from equations (15) and (20). For a short interval of time after initial contact dv/dt = z">o. Consider the vertical motion of the top of the ball, this being typical of the motion of all particles in the free portion of the pellicle. The downward acceleration is given by

$$-\frac{d^2}{dt^2}[R\ (2 - z)] = Rz''$$

Thus the **free** portion **accelerates** Initially. This is due to the decrease in the mass of the moving part of the pellicle, the retarding force having not built up sufficiently to decelerate this part of the pellicle in these early stages of deformation. A typical graph of the speed of the top of the ball against time up to the instant of maximum area of contact is shown in Fig 5 for an impact speed of 90 ft/sec (using the third approximation). (Naturally at this stage, this answer should be viewed with caution!)

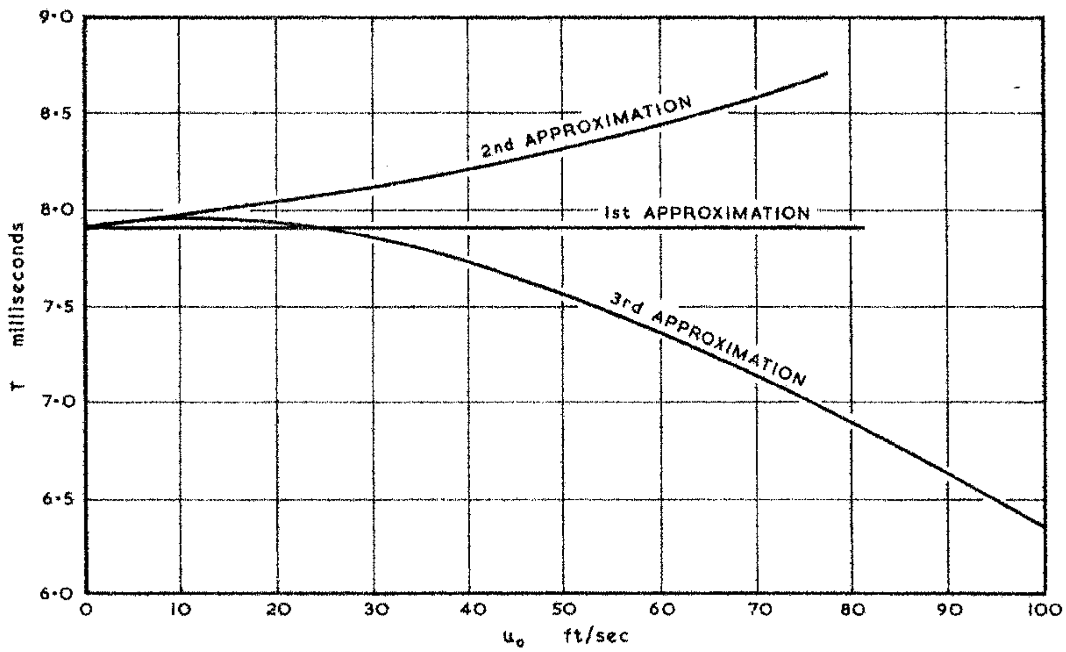

Fig 4 Time of Contact vs Impact Speed for the three theoretical approximations

EXPERIMENTAL RESULTS OVER A LIMITED RANGE OF U_0

Two sets of tests were performed in order to measure the impact speed, rebound speed, area of contact and (for the second set) the time of contact for a Mitre Multiplex Football inflated to 15 Ib/in^2

The first series of tests consisted in projecting the ball at a small angle to the vertical against a horizontal board which had been given a thin covering of chalk. The event was illuminated by means of a stroboscopic light (frequency 2500 flashes/min) and recorded by exposing the film of a stills camera. A vertical background grid was used to provide a reference for measurements. The path of the ball before and after impact could be seen from the developed film and the impact and rebound speeds calculated easily from measurements taken from the film. The area of contact was measured from the dimensions of the imprint left on the board. The results are included on Fig 3 alongside the theoretical curves.

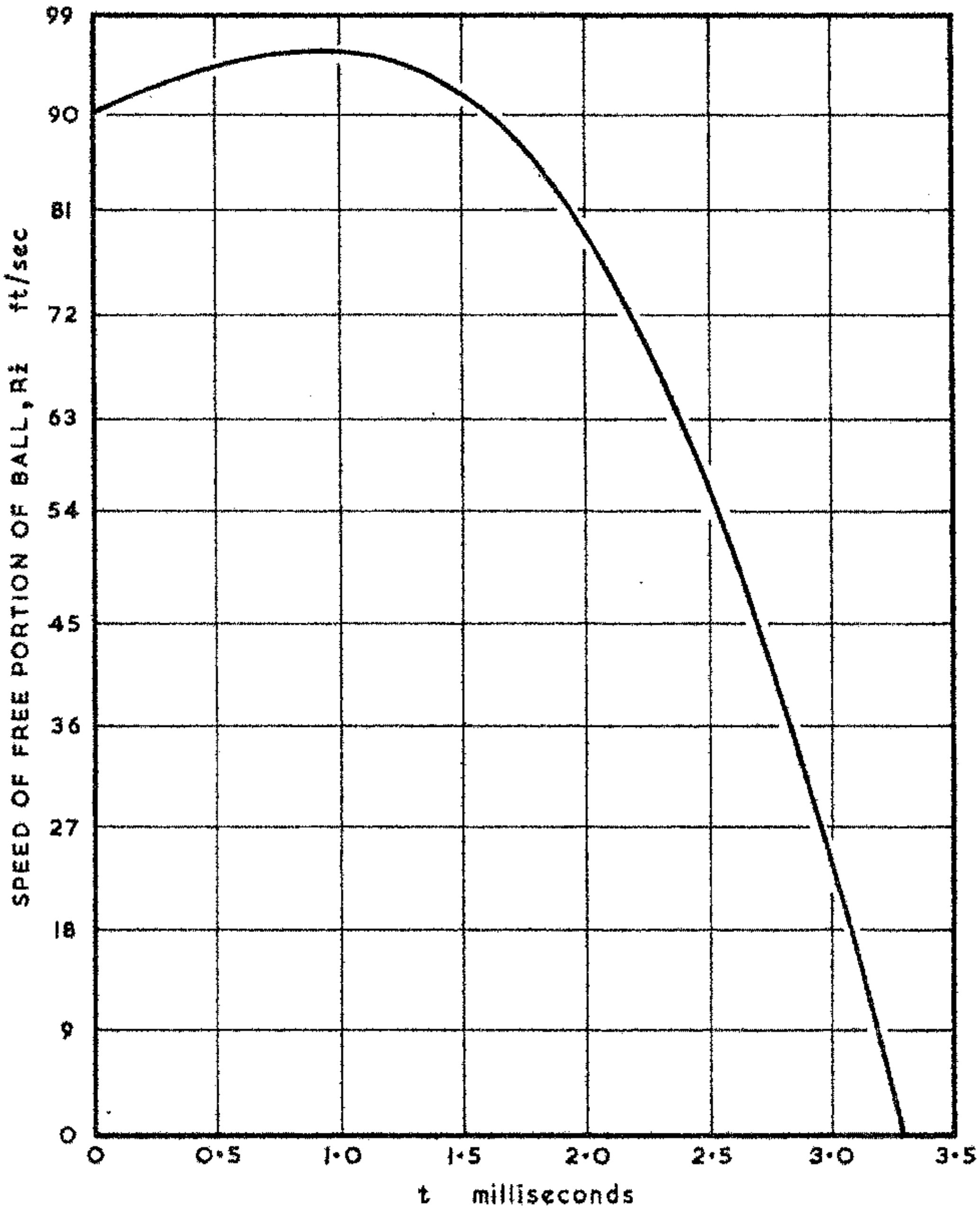

Fig 5 Speed of free portion of pellicle vs time up to instant of maximum area of contact for u_0 - 90 ft/sec.

In the second series of tests the time of contact between the ball and a rigid plate was measured using the apparatus which is shown diagramatically in Fig 6. As the ball touched each trip wire and the base plate, resistors were "shorted-out" thus giving step-wise increases in voltage. From the oscilloscope trace of this voltage the time interval between the ball touching the second and third trip wires and the duration of the contact with the base plate could be measured. As the separation between trips 2 and 3 was known a set of impact velocity/time of contact data was obtained. Again the range of the impact velocity was somewhat restricted owing to the provisional nature of the experiments but the trend of the relationship between the two parameters was quite clear from the tests. A set of values of impact speed and contact time is shown in Table II.

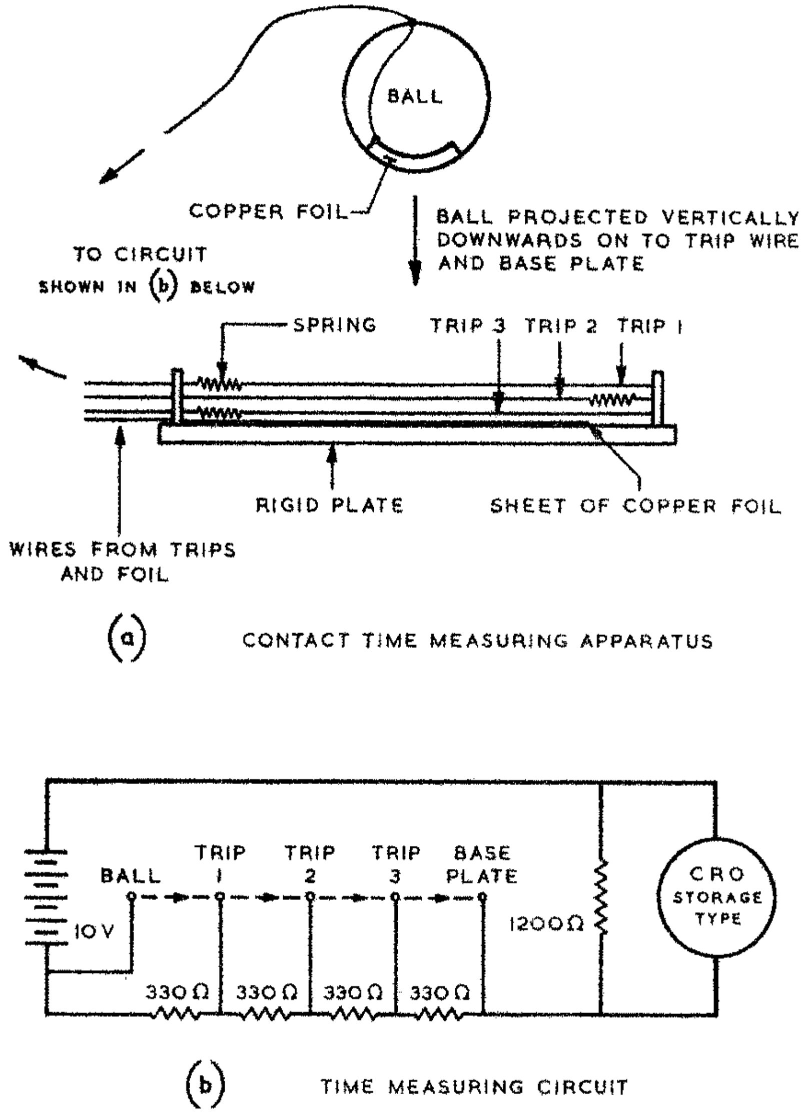

Fig 6 Simple arrangement for measuring time of contact and impact speed

A more detailed examination of contact phenomena will be the subject of a future investigation.

COMPARISON BETWEEN THEORIES AND EXPERIMENTS

Relationship between u_0 and A_m

It is clear from Fig 3 that, whilst there is good agreement particularly with the third approximation, the area of contact corresponding to a given impact speed indicated by the theory is consistently higher than that measured experimentally. This effect is due to the neglect of all sources of energy loss… in the theoretical model. Not all of the kinetic energy of the ball is used to compress the air inside the ball and so the value of Am which provides a measure of this compression is necessarily smaller than the theoretical value.

Relationship between u_o and T

The decrease in contact time with increasing impact speed which the third approximation predicts is borne out by the limited experimental results and it is clear that the increase of internal pressure of the ball during contact is an important factor in the contact behaviour.

u_o (ft/sec)	8.8	9.4	12.8	13.6	14.2	15.3	19.1	20.4	24.7
T (millisec)	8.3	8.2	8.0	8.0	7.8	7.6	7.6	7.6	7.5

TABLE II

It would appear that the decrease in the contact time starts at a lower impact speed than predicted theoretically. This could be due to the neglect of bending effects in the casing which are comparable with the contribution of the contact pressure. The presence of the copper foil could also have a significant effect at these lower impact speeds and yet another feature of the football which would have a bearing on this behaviour is the presence of the seams which provide local portions of increased stiffness. In spite of the neglect of these physical asperities and the material properties of the football the theory still appears to provide an adequate model for the impact behaviour.

BALL DEFLECTION OR 'BREAK'

Limitations of space prevent detailed discussion of interesting effects at impact with a rigid surface by a rigid sphere which possesses a velocity not wholly normal to the plane of impact and which also spins about an arbitary axis. Attention is directed to one feature and one striking result. A sphere which moves in a given vertical plane and whose axis of spin at impact is perpendicular to it, leaves the surface at an angle different from that at which it impinges; however, it continues its flight in the same vertical plane see eg S L Loney p274 [8]. Adapting Loney's elementary treatment it may be shown that if the spin axis is not perpendicular to the plane of flight prior to impact, then its vertical plane of flight after impact will be inclined. The magnitude of this angle of deflection depends on the velocity and angular velocity at impact as well as the assumed condition between the plane and the ball. It may easily be shown that for impact against a rough plane, a solid sphere may be deflected or 'break' through as much as about 42° and a spherical shell through 23° when forward motion of the ball persists.

Whilst there is some interest in discussions of the 'break' of a football at impact, yet this topic is of most interest in relation to cricket and table tennis, and much insight into bowling practices in cricket may be had by carefully examining the rigid body mechanics of plane impacting, spinning. rigid spheres.

PART II. ASPECTS OF THE FLIGHT OF A FOOTBALL

In flight, a football is subjected to gravitational force and to aerodynamic lift and drag forces. Attention will be directed towards applying published data on the lift and drag forces on a sphere to the specific example of a football in order to show how the characteristics of these forces influence the flight. Much of the basic information regarding these forces, in particular the experimentally determined dependence of the lift and drag coefficients on the Reynolds number, can be found in [1]. To illustrate the types of calculation which can be made, two features of the flight of a football will be examined,

(a) a set of speed-distance-time curves will be plotted for the horizontal flight of a non-spinning foot ball.

(b) an estimate will be made of the amount of swerve produced by the "lift" force exerted on a spinning football and it will be shown that this estimate is more than adequate to confirm that a goal can be scored direct from a corner kick.

(a) Horizontal flight of a non-spinning football

Provided the football is not spinning, the major aerodynamic force on it is the drag force. (There are random fluctuations in the aerodynamic force transverse to the direction of flight (2) -which could account for the wobbling sometimes noticed in the high speed flight of a football but these are effects secondary to the main decelerating drag force considered in this section.) The complex aerodynamic conditions which prevail immediately after the kick whilst the wake is building up behind the ball are ignored, For present purposes the wake is considered to be fully developed from the moment of impact and thus experimental data pertaining to steady state conditions can be used, Data on the aerodynamic forces exerted on a sphere are usually quoted in terms of non-dimensional force coefficients. The drag coefficient C_D is defined by

$$C_D = \frac{D}{\frac{1}{2}\rho u^2 A'} \qquad (21)$$

where D is the drag force, A' the cross-sectional area of the sphere, u its speed relative to the air and ρ is the density of the air, Within the usual range of football speeds it has been shown experimentally that C_D is a function only of the Reynolds number

$$Re = \frac{ud}{\nu}, \qquad (22)$$

d being the diameter of the sphere and ν the kinematic viscosity of the air. Typical plots of C_D versus R_e, taken from (7), are shown in Fig 7 for a set of smooth axially symmetric bodies including the sphere, The subsequent calculations wiIl be based upon the data given in Fig 7 although the surface of a football cannot always be described as smooth and certain materials used in modern footballs have a matt finish which could distinctly modify the results given below, The effects of surface roughness and surface irregularities will be discussed later.

The principal feature of Fig 7 is the existence of a critical Reynolds number of approximately 2.15×10^5 at which there is a sudden fall in C_D, Thus spheres travelling at slightly supercritical Reynolds numbers experience less drag than at subcritical ones.

Speed-distance-time curves

The equation of motion for a ball through stationary air and in the absence of all horizontal forces except the drag force D, is

$$D = -Mf \qquad (23)$$

where M is the balf mass and f the linear acceleration, Now from equation (21) ,

$$D = \tfrac{1}{2} C_D \rho u^2 A' \qquad (21i)$$

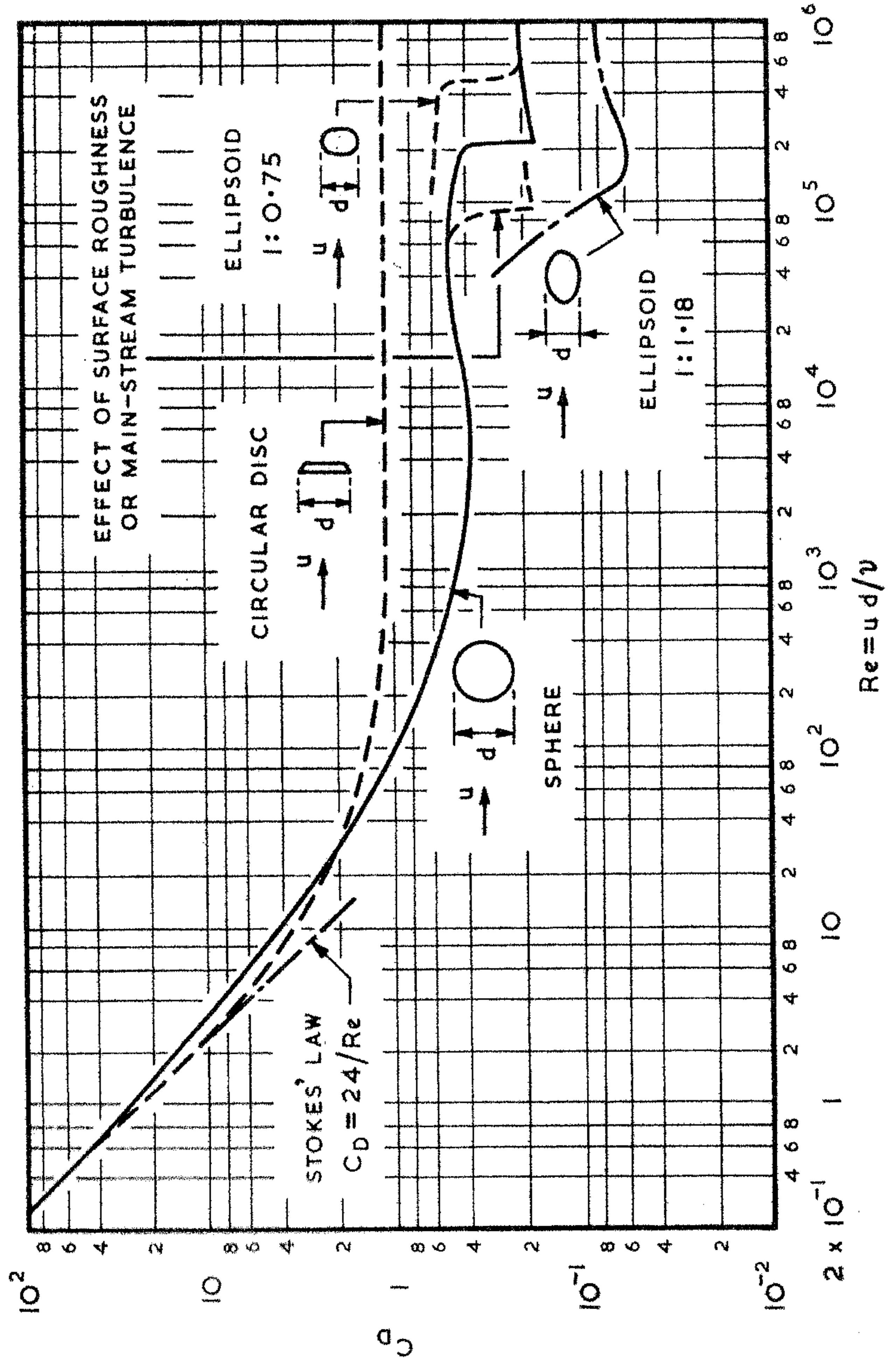

Fig 7 Drag coefficients of smooth axially-symmetric bodies. (Taken from [7])

where C_D is the overall drag coefficient, A' (the presentation area of the ball) is $\pi d^2/4$ and u is the ball speed. The drag coefficient C_D has to be determined experimentally but for a **smooth** sphere this is fortunately available and its variation with Reynolds number Re is shown in Fig 7.

The Reynolds number in air of a smooth sphere whose diameter d is that of a football ($8^1/_2$ in.) is

$$Re = \frac{ud}{\nu} = 4.44 \times 10^3 u$$

where ν, the kinematic viscosity of the air at a temperature of 20°C and a pressure of one atmosphere, is 16×10^{-5} ft^2/sec.

It is possible now to proceed numerically. For selected values of u, values of Re are obtained using the above value for ν and corresponding values of C_D are read from Fig 7. Thus with the drag force known for a sequence of values of u, the retardation undergone by a football of known mass can be found, and hence the distance covered in certain times for a ball moving with a given speed u_1 in the first place. This procedure is evidently tedious but, fortunately, when Fig 7 is examined it is seen that

(i) C_D is nearly constant over the range $10^3 < Re < 2.15 \times 10^5$,
i.e. for $0.23 < u < 50.0$ ft/sec it has a value of 0.4

(ii) C_D is nearly constant for $Re > 2.15 \times 10^5$,
i.e. for u > 50 ft/sec it has a value of 0.2. When $Re = 10^6$, U = 232 ft/sec.

With (i) and (ii) in mind it is thus possible to arrive at analytical expressions connecting s, t and u. From equation (23),

$$M \frac{du}{dt} = \frac{-C_D \rho A' u^2}{2} \tag{24}$$

$$\text{or } Mu \frac{du}{ds} = \frac{-C_D \rho A' u^2}{2}, \tag{25}$$

where s denotes linear distance and t is time. From equation (24)

$$\int_{u_1}^{u} \frac{du}{u^2} = \frac{-C_D \rho A'}{2M_0} \int_0^t dt.$$

U_1 is the speed at t = 0 and u the speed after time t, Hence

$$\frac{1}{u} - \frac{1}{u_1} = \beta t, \qquad (26)$$

where $\beta = C_D \rho A'/2M$.

From equation (25)

$$\int_{u_1}^{u} \frac{du}{u} = \frac{-C_D \rho A'}{2M} \int_0^s ds,$$

where s is the distance covered when the speed has fallen from U_I to u.

Hence

$$\log_e \frac{u_1}{u} = \beta s. \qquad (27)$$

Now ρ the density of the air at N.T.P. is 0.00233 slugs/ft^3 (specific weight is 0.0765 Ib/ft^3 and for the two ranges,

$$\begin{aligned} &\text{(i)} \quad \beta = C_D \rho A'/2M = 5.87 \times 10^{-3}\ ft^{-1} \text{ and} \\ &\text{(ii)} \quad \beta = 2.94 \times 10^{-3}\ ft^{-1}. \end{aligned} \qquad (28)$$

Table 3 gives values of t and s for specific initial values of u in each of the two ranges, whilst Fig 8 shows graphically how u_1 ,u, t and s are related, the results for the two ranges having been combined. If it is required to find how long it takes a football to travel 76 ft (ie. approximately a 25 yard 'drive') starting from a speed u_1 = 68 mile/h, then on Fig 8 look up the value of s at 100 ft/sec or 68 mile/h which is 532 ft, and then subtract 76 to give 456 ft, and the value of u after this distance is 80 ft/sec or 54.5 mile/h. The difference between the times at these two speeds, Le. (3.41 - 2.55) ,= 0.86 sec, is the time taken for the ball to travel this distance having started at t 100 ft/sec. For a 36 yard 'drive' starting at 110 ft/sec or 75 mile/h, the time required is 1.16 sec and the speed falls to 80 ft/sec or 54.5 mile/h at the end of this distance.

Also shown on Fig 8 is the force retarding the ball at each speed level obtained using equation (21). For the two ranges

(i) $D = 1.84 \times 10^{-4} u^2$ lbf and (ii) $D = 0.92 \text{ X } 10^{-4} u^2$ lbf.

It is very interesting to note that because of the sudden fall in C_D at Re = 2.15×10^5, see Fig 7, ie. at the end of range (i} at about 50 ft/sec (34 mile/h), the resistance to the flight of the football decreases despite its increase in speed, until the speed exceeds about 70 ft/sec (48 mile/h).

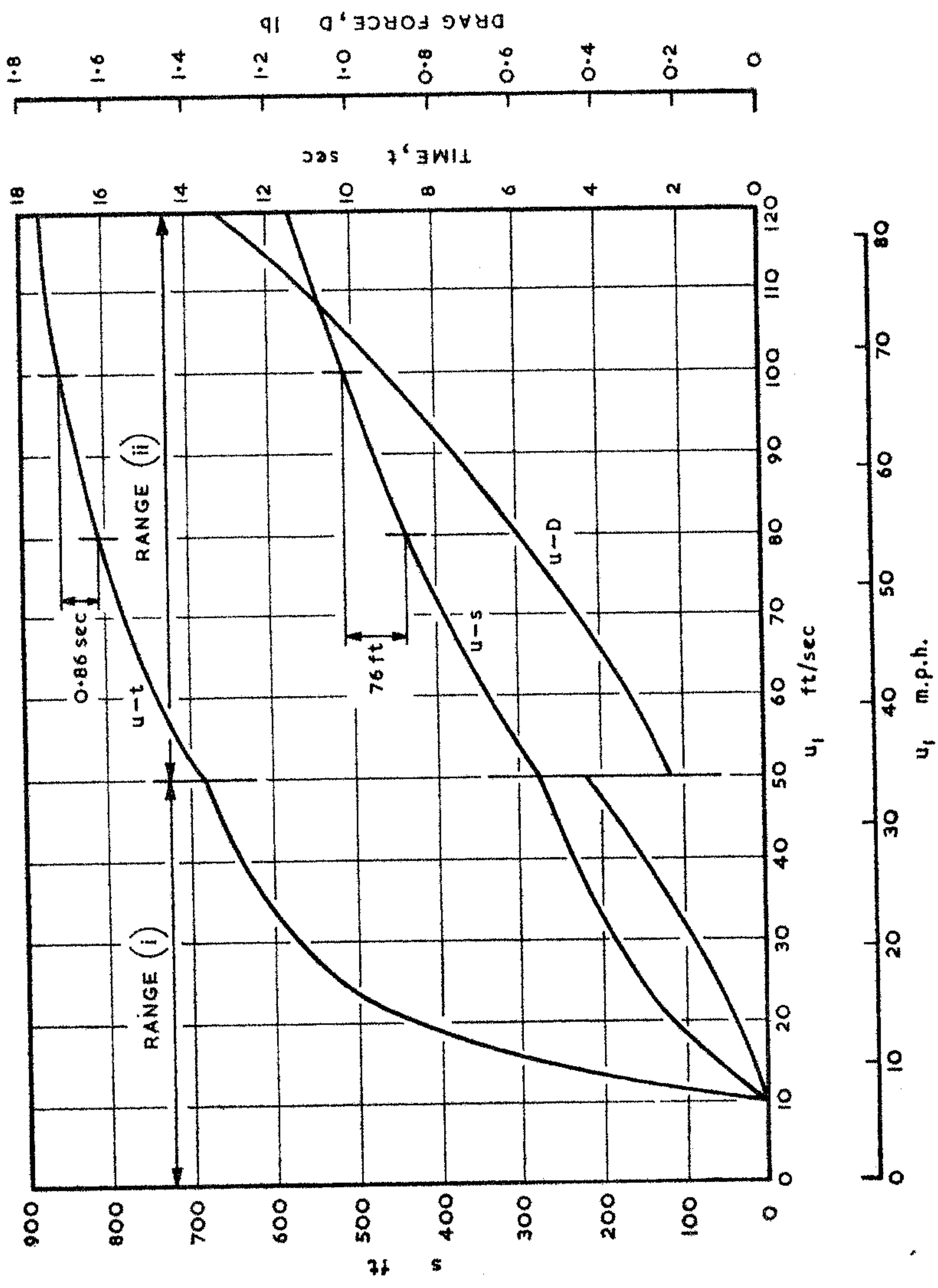

Fig 8 Speed - time - distance - drag force curves for football in horizontal flight

TABLE 3

	u_1		t	s	D
	ft/sec	mile/h	sec	ft	lbf
RANGE (i)	10	6.8	0.00	0	0.02
	20	13.6	8.53	118	0.07
	30	20.4	11.36	187	0.17
	40	27.3	12.80	236	0.29
	50	34.1	13.66	274	0.46
RANGE (ii)	50	34.1	0.00	0	0.23
	60	40.4	1.13	62.1	0.33
	70	47.7	1.94	115	0.45
	80	54.5	2.55	160	0.59
	90	61.3	3.02	200	0.75
	100	68.1	3.41	236	0.92
	110	74.9	3.71	268	1.11
	120	83.4	3.96	298	1.32

It is curious that much of the game of football should be played in the one hydrodynamic region where there is a distinct critical Reynolds number. Footballers could thus be expected to develop one set of anticipations for playing balls up to a speed of, say, 40 ft/sec (27 mile/h) and another for above 70 ft/sec (45 mile/h) with an expectation which anticipates anomaly in the region in between; most football takes place, of course, in range (i).

Using the same critical Reynolds number it is interesting to note how the corresponding ball speed u_0 changes with atmospheric temperature, so that at 7°C, which represents a cold day in winter, u_c = 45 ft/sec, and at 34°C, a hot summer's day, U_c = 55 ft. sec.

The Stockport Express for 26 October 1972 reports shots in which "using a strobo light technique" the ball reached 73.3 mile/h; reports of ball speeds of up to 75 mile/h (110 ft/sec) are now common.

INFLUENCE OF SURFACE FINISH

The drag on a sphere is a result of the non-symmetric pressure distribution resulting from the presence of the wake which arises out of the separation of the boundary layer from the surface of the sphere. For the same air speed, the further around the sphere that separation occurs, the smaller is the pressure defect and consequently the smaller is the drag force.

The position of the point of separation for bluff or rounded bodies depends on phenomena in the boundary layer adjacent to the ball surface so that, for example, vorticity in the approaching fluid is very important. When the flow in the boundary layer becomes turbulent before reaching a separation point, a shift towards the rear is promoted causing the resistance to be lowered very considerably. A classic experiment due to Prandtl (see p40 [4]) demonstrates how a hoop of thin wire (1/300 that of the sphere diameter) when fixed to a sphere ahead of the point where the flow would separate if it were laminar (normally this is about 80° from the foremost point of the body) acts so as to set up eddies in the boundary layer and to displace the separation point backwards to the 110° to 120° position thus substantially diminishing the resistance. Turbulence in the boundary layer inhibits separation which also explains the existence of the fall in C_D as the Reynolds number reaches its critical value, since turbulence sets in at higher air speeds. The particular Reynolds number at which the boundary layer becomes turbulent is influenced considerably by the surface finish of the sphere. Roughening the surface causes an onset of turbulence at a lower air speed than for a smooth sphere and at some speeds the drag on a roughened sphere is only about 1/5th the drag on a smooth sphere. This behaviour is summarised in Fig 7. Thus the effect of increase in surface roughness is to lower the Reynolds number at which the critical drop in C_D occurs; the fall in C_D become smaller however and the supercritical C_D larger. Coarse roughness causes the disappearance of the fall in C_D and it becomes independent of Reynolds number. The waviness of a smooth surface is found also to lead to somewhat similar results though, in some cases erratic results are encountered.

In respect of footballs there is obviously both waviness and roughness. There is systematic waviness due to stitching and roughness due to the nature of the material of which the ball is manufactured. The newer materials of which footballs are made - laminated layers of cloth and rubber, some with smooth and others with matt finishes-may certainly give rise to different aerodynamic performances as against the older style leather balls; even a new leather ball behaves differently from one which is well worn.

These two aspects of the effect of surface detail on drag are also characterised in the case of a cricket ball; the expectations from a new ball in cricket are, on the one hand, proverbial, whilst the consequences of the seam of the ball in bowling have been the object of identical remarks by Royle [6] and Lighthill [4]. The latter has written, "A seam bowler in cricket practises the art of tripping the boundary layer on one side of the ball only, so as to produce an asymmetric pressure distribution that will make the ball swerve". In the same vein, it may be imagined that the lacing on an old style football would seem to have aerodynamic consequences.

(b) The lift of a rotating football

There is a paucity of information about the lift and drag coefficients, C_L. (equal to $L^1/_2\rho u^2A'$ where L is the lift force) and C_D, for a rotating sphere, but using Maccoll's results [2] some interesting calculations may be made. Maccoll measured the forces on a sphere of diameter 6 in. when $Re \simeq 10^5$ along a cross-wind axis in a wind tunnel and his results are reproduced in Fig 9. He showed that, approximately, when the ratio e of equatorial speed to wind speed > 1, the lift coefficient C_L is about 0.35, when e = 0.4, C_L is slightly **negative**, and for 0.4 < e < 1, C_L increases linearly from zero to 0.35. The aerodynamic effect of combining translational and rotational motion is clearly a complicated one.

For simplicity, by assuming a football in horizontal flight, the "bending" or "curving" undergone by the ball in the particularly important case of the "taking of a corner" may be estimated. Similar calculations apply, of course, to any "banana" shot. No direct experimental evidence is to hand to facilitate comparison with theory, beyond the well known qualitative fact that a goal may be scored directly from a corner kick and that a considerable amount of swerve can be imparted to a ball by kicking it off-centre, thereby making it spin.

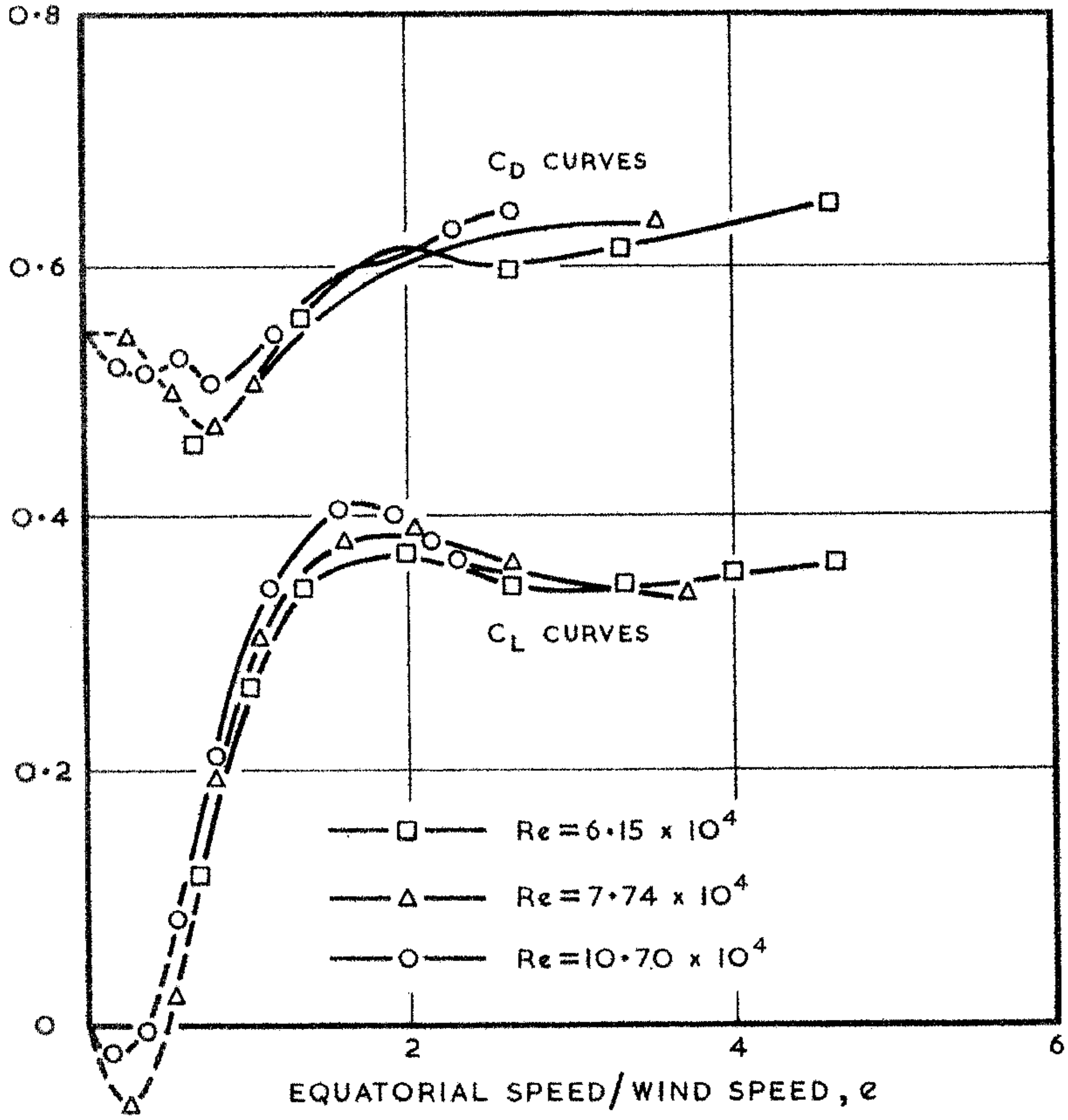

Fig 9 Variations of Drag and Lift Coefficients with Reynolds number for a Spinning Sphere. (Taken from [1]).

Suppose that when the ball is kicked it is given a rotational speed of 15 rev/sec about a vertical axis and that it may be assumed to have an **average or constant** horizontal speed of 60 ft/sec during its flight to the goal-mouth, then e is $\simeq$ 0.6 and Fig 9 shows C_l, $\simeq$ 0.1. Thus the horizontal side force is $C_L \rho A'u^2/2$ and the transverse acceleration is $C_L \rho A'u^2/2M$. If the distance from the Corner flag to the goal is 120 ft, then in 120/60 = 2 sec, the in-swing of the ball from its initial line of flight is $^1/_2 f.2^2 = C_L \rho A^T u^2/M \simeq 10$ ft.

It should be noted from Fig 9 that negative lift coefficients may be encountered ("an out swinger") if the rate of spin in the horizontal plane is insufficiently large.

Now, assuming uniform acceleration, the transverse speed of the ball when it reaches the vicinity of the goal-mouth at this time is f.t = 5. 120/60 =, 10 ft/sec, so that in travelling across the goal-mouth, ie. through a distance of 24 ft at 60 ft/sec, the transverse displacement is approx 10 x 24/60 = 4 ft -- sufficient to cause the ball to enter the net in an undoubted fashion.

ACKNOWLEDGMENTS

The authors would like to express their gratitude to Dr. J H Gerrard of the Department of Mechanics of Fluids in the University of Manchester for supplying information regarding the literature on lift and drag forces on a sphere, and especially to Mitre Sports Limited, Huddersfield, for their kind donation of footballs with which to conduct the impact tests. Thanks are also due to Mr. W. Griffiths of Manchester City A.F.C. for the gift of footballs with which a set of preliminary experiments was performed.

They would also like to thank Professor David Newland and Mr Philip Mayne for their comments on the manuscript and for their careful checking of some of the calculations.

REFERENCES

[1] J S GOLDSTEIN, Modern Developments in Fluid Dynamics, Vols I and 11, Oxford University Press, (1938).

[2] W W WILMARTH and R L ENLOW, J Fluid Mech, 36 part 3, 417 (1969)

[3] H SCHLICHTING, Boundary Layer Theory, McGraw-Hill, (1968)

[4] M J L1GHTHILL, Osborne Reynolds and Engineering Science Today, Proceedings of the Osborne Reynolds Centenary Symposium, University of Manchester, 1968, Manchester University Press, (1970)

[5] D P UPDIKE and A KALNINS, J Appl Mech, 37 No 3, 635, (1970)

[6] J K ROYI.E·~Prlvate communications

[7] B S MASSEY, Mechanics of Fluids, D Van Nostrand, London, (1968)

[8] S L LONEY, Dynamics of a Particle and of Rigid Bodies, CUP, 1953, p274